Government spending and fiscal policy stabilizing rules*

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Abstract

Considering a finance constrained economy, we discuss the stabilization role of cyclical labour/capital income tax rates, under a balanced budget rule, in an environment where indeterminacy and sunspot fluctuations prevail due to consumption externalities and constant structural public expenditures, financed by income taxation.

We find that sufficiently procyclical labor and capital income specific tax rates are able to stabilize locally the economy, eliminating business cycles driven by self-fulfiling prophecies that stay arbitrarily close the steady state. However, procyclical specific tax rates lead to steady state multiplicity and, whenever the steady state under analysis becomes a saddle path, there is at least another steady state with a lower level of output, that is either a source or indeterminate and Hopf bifurcations may occur. Hence, depending on expectations, the economy may end up converging to a a lower level of output and it is not completely insulated from instability linked to volatile expectations.

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1 Introduction

Fiscal policy can contribute to macroeconomic (in)stability mainly through two channels. First, a considerable fraction of public spending reflects government commitments which are independent of the business cycle. Second, governments can deliberately use public spending and tax instruments to offset business cycles fluctuations. However, there is not an integrated study of these two types of fiscal policies: those that introduce instability because of the need to finance a minimum of public expenditure (generating countercyclical tax rates)[1] and those that can promote stability.[2] Moreover, governments use several tax instruments and tax differently labor and capital incomes. Nevertheless the comparison of the cyclical properties of labor and capital income taxes able to bring saddle path stability, in an economy where indeterminacy would prevail in their absence, is a question not yet addressed in the related literature. Our work fills these gaps.

We consider a one sector economy where equilibrium indeterminacy and expectations driven fluctuations emerge due to (i) the existence of constant government spending commitments financed by (countercyclical) income taxation and (ii) the presence of consumption externalities. In this context, we introduce cyclical specific tax rates on labour and capital income and discuss which are the cyclical properties of these fiscal rules that are able to stabilize the economy eliminating business cycles fluctuations.

We find that sufficiently procyclical tax rates on capital and/or labor income (and therefore procyclical government spending) bring local saddle path stability. These results confirm the insights of previous research, according to which procyclical labor tax rates promote determinacy. However, our finding that sufficiently procyclical capital taxation stabilizes locally the economy is novel, supporting also the traditional Keynesian view on taxation,[3] and rehabilates the role of capital income taxation as a stabilization instrument. We also find that when labor(capital) income tax rates are sufficiently procyclical, saddle path stability can be achieved with flat or even countercyclical capital(labor) income tax rates. This result, which indicates that labor and

[1] Schmitt-Grohé and Uribe (1997) Pintus (2004)and Gokan (2006) have shown that countercyclical tax rates on income used to finance a constant flow of government spending have a destabilizing effect on the economy by triggering expectations-driven cycles due to equilibrium indeterminacy and bifurcations.

[2] Guo and Lansing (1998) and Guo (1999) show that progressive income or labor taxation bring saddle path stability and eliminate indeterminacy in an environment where the latter would prevail due to increasing returns to scale.

[3] According to standard Keynesian models the government should lower (increase) tax rates in bad (good) times in order to stabilize the business cycle, reducing the possible costs of fluctuations. On this issue see also Ljungqvist and Uhlig (2000).

capital taxation can be seen as substitutable stabilization tools, shows that governments, whose aim is to stabilize the economy, do have a choice among different combinations of procyclical and countercyclical labor and capital taxation. This is a new result and validates the current policy debate on how the tax burden should be divided between labor and capital income. However there is danger with procyclical tax rates on labor and/or capital income, since at least two steady states coexist and local stabilization exercises may become futile indeed. In fact, even if a sufficiently procyclical tax rate is able to make the steady state, under study, locally saddle stable, we show that there will be another steady state, with a lower level of output (which may be indeterminate) and global stability is not obtained.

In the present economic crisis European peripheric countries are struggling to balance the public budget, imposing cuts in government spending (procyclical government spending). At the same time, many analyts recomend a cut in tax rates on labor and capital income. Our results show that, although these policies may be able to reduce local instability they do not erradicate the possibility of a deeper crisis and further instability, due to stronger pessimistic selfulfilling expectations about future income, that will shoft the economy to the lower activity steady-state.

The rest of the paper paper is organized as follows. In the next section we present the model considered, obtain the perfect foresight equilibria and define the steady state. In section 3 we prove steady-state existence. We then study the local stability properties in section 4, stating conditions for the degree of cyclicality of the tax rate under which the steady state is a sink, a source or a a sadlle. In section 5 we discuss the role of specific tax rates on capital and labor income as stabilization instruments. Steady state multiplicity and global stability are adressed in section 6 and we discuss our results in Section 7. Finally some concluding remarks are provided in section 8.

2 The Model

The model here considered extends the Woodford (1986)/Grandmont et al. (1998) framework introducing public spending financed by taxation and two different types of externalities in preferences: structural government spending and consumption externalities. We consider a perfectly competitive monetary economy with discrete time $t = 1, 2, ..., \infty$ and heterogeneous infinitely lived agents of two types: workers and capitalists. Both consume the final good, but only workers supply labor. There is a financial market imperfection that prevents workers from borrowing against their wage income and workers are more impatient than capitalists, i.e. they discount the future more than the latter. So, in a neighborhood of a monetary steady state, capitalists hold the whole capital stock and no money, whereas workers save their wage earnings through money balances and spend it in consumption in the following period. The final good, which can be used for consumption pr capital investment, is produced by firms under a Cobb-Douglas technology characterized by constant returns to scale. We introduce two types of public spending in this framework: constant structural government spending that reflects commitments independent of the business cycle, and cyclical government spending whose value may vary with business cycles. The later is considered "wasteful"public spending and is financed by specific variable labor and/or capital income taxes. In contrast, structural spending is financed by income taxation and has utility, i.e., we introduce structural governement spending positive externalities on preferences. Finally, we also consider consumption externalities, i.e., we assume that the individual utility of consumption is affected by the current consumption of other similar agents. More precisely, we consider that private consumption and government services are non-separable and Edgeworth complements, following Ni (1995) who provides empirical support for these assumptions. In the case of consumption externalities we assume that the marginal utility of an individual own consumption increases with aggregate consumption.[4] This feature is referred in the literature as the desire to keep up with the Joneses. Some authors have analyzed the theoretical impact of this type of externalities on economic growth and optimal tax policy.[5] However, to our knowledge, this is the first time that the interrelations beetween 'keeping up with the Joneses' preferences, wastefull government spending financed with cyclical labor and/or capital income tax schedules, a constant amount of structural government spending that affects positively utility, financed by (countercyclical) income taxation and macroeconomic (in)stability are analyzed jointly from a policy point of view. The detailed description of the model and perfect foresight equilibrium is provided below. In order to focus our analysis on instability linked to autonomous volatility on expectations, we disregard uncertainty about the economic fundamentals, considering stationary preferences, technology and fiscal policy rules.

[4]See Gali (1994.)

[5]See Ljungqvist and Uhlig (2000), Guo (2005) and Wendner (2010).

2.1 Production

In each period $t = 1, 2, ..., \infty$, both capital $k_{t-1} > 0$ and labor $l_t > 0$ are used to produce output y_t under a Cobb-Douglas technology with constant returns to scale,

$$y_t = k_{t-1}^s l_t^{1-s} \tag{1}$$

where $s \in (0,1)$ represents the capital share of income. From profit maximization, the marginal productivities of capital and labor are respectively equal to the real rental rate of capital (i.e. the real interest rate) ρ_t and the real wage ω_t, i.e.

$$\rho_t = s k_{t-1}^{s-1} l_t^{1-s} \tag{2}$$

$$\omega_t = (1-s) k_{t-1}^s l_t^{-s}. \tag{3}$$

There are no economic profits at equilibrium and therefore $y_t = \omega_t l_t + \rho_t k_{t-1}$.

2.2 The Government

As usually assumed in the literature the government runs a balanced buget. The main novelty is that we consider two types of expenditures (and revenue). On one hand, the government collects every period, through income taxation, a constant amount of fiscal revenue, used to finance an equivalent flow of government expenditures, $\overline{G}$. Therefore, letting $\tau_{y_t} \in (0,1)$ denote the income tax rate, we have:

$$\tau_y(y_t) = \tau_{y_t} \equiv \frac{\overline{G}}{y_t} \tag{4}$$

Note that the tax rate τ_{y_t} is countercyclical, decreasing (increasing) when output increases (decreases) with an elasticity of -1. The level of spending, $\overline{G}$, reflects the views of government and society on the appropriate size of unavoidable government expenditures, that should remain constant along business cycles. We further assume that this amount of spending corresponds to public services that positively influence households' utility.

On the other hand, we also introduce possible wasteful variable government expenditures that are financed by specific labor and/or capital income taxes, according to fiscal policy rules whose aim is to stabilize the economy. Accordingly, these tax rates vary with the level of income/output in the economy, being taken as given by individuals. Hence, the specific tax rates on labor income $\tau_{Lt} \in [0,1)$ and on capital income $\tau_{Kt} \in [0,1)$, at period t, are given respectively by

$$\tau_L(y_t) = \tau_{Lt} \equiv \mu_L y_t^{\phi_L} \quad \text{and} \quad \tau_K(y_t) = \tau_{Kt} \equiv \mu_K y_t^{\phi_K} \tag{5}$$

The parameter $\phi_i = \frac{d\tau_i}{dy}\frac{y}{\tau_i} \in R$ for $i = L, K$, represents the elasticity of the tax rate with respect to total income or output. When $\phi_i < 0$ the tax rate decreases when the level of output expands, i.e., the tax rate moves countercyclically. The case of $\phi_i > 0$ corresponds to the case where the tax rate increases with output, i.e. the tax rate is procyclical. For $\phi_i = 0$ the tax rate is constant, so that cyclicality of tax rates is absent.[6] The parameters $\mu_L \in (0,1)$ and $\mu_K \in (0,1)$ represent the tax rates when $y = 1$.

Summing up, there are taxes on income used to finance a constant flow of government expenditures $\overline{G}$ and, on top of that, the government can tax capital and labor income differently, keeping the budget balanced. Accordingly, we have:

$$G_t = \tau_L(y_t)\,\omega_t l_t + \tau_K(y_t)\,\rho_t k_{t-1} + \overline{G} \tag{6}$$

Note that we distinguish between cyclical public expenditures $(G_t - \overline{G})$, whose amount may vary along the business cycle, and structural government spending $\overline{G}$, that does does not respond to economic fluctuations.[7] This seems to be better suited to deal with current concerns of countries undergoing problems of sovereign debt, where the appropriate size of government has also been under discussion.

For future reference we introduce the following notation:

$$a_L(y) \equiv \phi_L \frac{\tau_L(y)}{1-\tau_L(y)} \in (-\infty, +\infty) \tag{7}$$

$$a_K(y) \equiv \phi_K \frac{\tau_K(y)}{1-\tau_K(y)} \in (-\infty, +\infty) \tag{8}$$

$$b_L(y) \equiv \frac{1-\tau_L(y)}{1-\tau_L(y)-\tau_y(y)} \in [0, +\infty) \tag{9}$$

$$b_K(y) \equiv \frac{1-\tau_K(y)}{1-\tau_K(y)-\tau_y(y)} \in [0, +\infty) \tag{10}$$

[6] This specification nests most cases considered in the literature. For example, the case considered in Gokan (2006), Pintus (2004) and Schmitt-Grohé and Uribe (1997) where a constant amount of public expenditures is financed by taxes corresponds to the case where $\phi_i = -1$ (as in (4)).

[7] Note that from (4) and (6), both structural and cyclical budgets are balanced. If we had assumed that the goverment only balances the total budget our results would not change significantly.

2.3 Workers

We introduce externalities in the consumption of workers. Consumption externalities correspond to the idea that the individual utility of consumption is affected by the current consumption of others (envy or altruism), so that aggregate or average consumption becomes an argument of the utility function.[8] Here we assume that individual workers compare their own consumption, $c_t^w \geq 0$, to the average consumption of workers, $\overline{c}_t^w$. We also introduce government spending externalities,[9] assuming that structural government expenditures are seen as useful public consumption that generates utility, so that total consumption utility depends also on $\overline{G}$.

Workers behave as if they decide each period t on the level of current labor supply l_t and of planned consumption for period $t+1$, c_{t+1}^w saving their wage income (net of taxes) on money holdings and using them to spend on consumption during the next period.

We consider that preferences of the representative worker over c_{t+1}^w and l_t are given by the following utility function:

$$U(c_{t+1}^w, \overline{c}_{t+1}^w, \overline{G}, l_t) \equiv c_{t+1}^w \left(\overline{c}_{t+1}^w\right)^\chi \overline{G}^\eta / B - l_t \tag{11}$$

with $l \in \left\{0, \widetilde{l}\right\}$, $\widetilde{l}$ is the worker's time endowment exogenously specified and possibly infinite, $B > 0$ is a scaling parameter, $\chi > 0$ represents the degree of private consumption externalities, and $\eta > 0$ represents the degree of (structural) public consumption externalities. The linearity of utility in hours worked follows the indivisible labor formulation of Hansen (1985) and Rogerson (1988). Note that $\chi > 0$ corresponds to the "keeping up with the Joneses" case, according to which the marginal utility of individual consumption is increasing in $\overline{c}_t^w$ ($\frac{\partial^2 U(x)}{\partial c^w \partial \overline{c}_t^w} > 0$).[10] Note also that $\eta > 0$, so that c_t^w and $\overline{G}$ are Edgeworth complements, i.e. the marginal utility of individual consumption is increasing in $\overline{G}$ ($\frac{\partial^2 U(x)}{\partial c^w \partial \overline{G}} > 0$).[11]

Budget constraints are given by

$$p_{t+1} c_{t+1}^w = m_t = (1 - \tau_L(y_t) - \tau_y(y_t)) w_t l_t \tag{12}$$

with $1 - \tau_L(y_t) - \tau_y(y_t) > 0$, w_t is the nominal wage at period t, m_t represents

[8] See Alonso-Carrera et al. (2008), Ljungqvist and Uhlig (2000), Weder (2000) and Gali (1994).

[9] See Seegmuller (2003), Utaka (2003) Cazzavillan (1996), Fernandez et al. (2004), Guo and Harrison (2007), Lloyd-Braga, Modesto and Seegmuller (2008) and Zhang (2000).

[10] Here we follow the specification considered in Gali (1994).

[11] Ni (1995) provides empirical support for Edgeworth complementarity between private and public consumption. However our results do not depend on η.

money holdings at the beginning of period $t+1$ and p_{t+1} represents the expectation for the price of the final good, which ends up being identical, under perfect foresight, to its market equilibrium level realized at $t+1$. Workers take as given tax rates, the average consumption of workers and structural public spending.[12] Hence, from maximization of utility given in (11) subject to the budget constraints (12), l_t and c^w_{t+1} are given by $p_{t+1}c^w_{t+1} = (1-\tau_L(y_t)-\tau_y(y_t))w_t l_t$, together with the intertemporal trade-off between future consumption and leisure:

$$c^w_{t+1}\left(\overline{c}^w_{t+1}\right)^{\chi}\overline{G}^{\eta}/B = l_t \tag{13}$$

We can see that labor is non predetermined variable, whose current value depends on the consumption level for $t+1$ planned by the worker at time t, which is influenced by expectations for p_{t+1}. Therefore, there is a priori room for fluctuations in employment and output driven by changes in expectations.

2.4 Capitalists

The representative capitalist maximizes the log-linear lifetime utility function $\sum_{t=1}^{\infty}\beta^t \ln c^c_t$ subject to the budget constraint $c^c_t + k_t = (1-\delta+(r_t/p_t)(1-\tau_K(y_t)-\tau_y(y_t)))k_{t-1}$, with $1-\tau_K(y_t)-\tau_y(y_t) > 0$ and where c^c_t represents his consumption at period t, k_t is the capital stock held at the end of period t by capitalists, $\beta \in (0,1)$ his subjective discount factor, r_t the nominal interest rate and $\delta \in (0,1)$ the depreciation rate of capital.[13] Capitalists also take the tax rate as given. Solving the capitalist's problem we obtain the capital accumulation equation:

$$k_t = \beta\left[1-\delta+(r_t/p_t)(1-\tau_K(y_t)-\tau_y(y_t))\right]k_{t-1}. \tag{14}$$

2.5 Equilibrium

Equilibrium on labor and capital markets requires $\omega_t = w_t/p_t$, $\rho_t = r_t/p_t$. Considering that $m>0$ is the constant money supply, at the monetary equilibrium, where $(1-\tau_L(y_t)-\tau_y(y_t))w_t l_t = m$ in every period t, we have $c^w_{t+1} = \omega_{t+1}(1-\tau_L(y_{t+1})-\tau_y(y_{t+1}))l_{t+1}$. Therefore:

[12] Since in our framework tax rates depend on aggregate variables (see (4), and (5)) individuals, being atomistic, take tax rates as given. A simar argument applies to average consumption of workers and structural government spending.

[13] We do not introduce consumption or government spending externalities into capitalists' preferences because, since they have a log-linear utility function, such externalities would not affect the dynamics nor the steady state.

Definition 1 *A perfect foresight intertemporal equilibrium is a sequence* $(k_{t-1}, l_t) \in R^2_{++}$*,* $t = 1, 2, ..., \infty$*, that, for a given* $k_0 > 0$*, satisfies*

$$[(1 - \tau_L(y_{t+1}) - \tau_y(y_{t+1}))\omega_{t+1} l_{t+1}]^{1+\chi} \overline{G}^\eta / B = l_t \tag{15}$$

$$k_t = \beta \left[1 - \delta + \rho_t (1 - \tau_K(y_t) - \tau_y(y_t))\right] k_{t-1} \tag{16}$$

with y*,* ω*,* ρ *given respectively by (1)-(3) and where* $\tau_y(y)$*,* $\tau_L(y)$ *and* $\tau_K(y)$ *are given respectively by (4), and (5) satisfying* $1 - \tau_L(y_t) - \tau_y(y_t) > 0$ *and* $1 - \tau_K(y_t) - \tau_y(y_t) > 0$*.*

Equations (15) and (16) represent, respectively, the intertemporal trade-off between consumption and leisure and capital accumulation. They determine the dynamics of employment and capital used in production for this economy through a two-dimensional dynamic system with only one predetermined variable, the capital stock k. Indeed, the amount of capital used in production at period t, k_{t-1}, is a variable determined by past actions. Employment l_t, on the contrary, is affected by expectations of future events as explained before.

2.6 Steady State

A steady state (k, l) **of** (15) and (16) is a stationary solution $k_t = k_{t-1} = k$ and $l_{t+1} = l_t = l$ of that dynamic system with y given by (1). Using (1)-(3), note that $\omega l = (1 - s)\, y$ and $\rho = s\,(y/l)^{(s-1)/s}$. Hence,

Definition 2 *A steady state is a pair* $(k_*, l_*) \in \Re^2_{++}$ *with the corresponding level of output* $y_* = k_*^s l_*^{1-s} \in \Re_{++}$*, such that*

$$H(y_*) = \overline{H}$$

$$l_* = \left[(1 - s)\, y_* (1 - \tau_L(y_*) - \tau_y(y_*))\right]^{1+\chi} \left(\overline{G}^\eta / B\right) \tag{17}$$

$$k_* = \left(y_* / l_*^{1-s}\right)^{1/s} \tag{18}$$

where

$$H(y) \equiv y_*^{\frac{1-s}{s}\chi} \left[1 - \tau_K(y_*) - \tau_y(y_*)\right] \left[(1 - s)\,(1 - \tau_L(y_*) - \tau_y(y_*))\right]^{(1+\chi)\frac{1-s}{s}} \tag{19}$$

$$\overline{H} \equiv \frac{\theta}{\beta s} \left[\frac{B}{(1 - s)^{1+\chi}\, \overline{G}^\eta}\right]^{\frac{1-s}{s}} > 0$$

with $\tau_y(y)$*,* $\tau_K(y)$ *and* $\tau_L(y)$ *given in (4)-(5),* $1 - \tau_K(y) - \tau_y(y) > 0$*,* $1 - \tau_L(y) - \tau_y(y) > 0$ *and* $\theta \equiv 1 - \beta(1 - \delta) \in (0, 1)$*.*

We ensure the existence of a steady state following the usual procedure of fixing the scale parameter at the appropriate level. See Section 4. The steady state may however not be unique. Later on in Section 6 we will discuss which cyclical properties of the specific tax rates generate multiplicity of steady states.

3 Local Stability properties

We now study the local stability properties of our dynamic system around a steady state y_* in terms of the cyclical properties of the tax rate on labor and capital income, here represented through the fiscal "variables" a_L and a_K, defined in (7)-(8). We first log-linearize the system (15)-(16) around the steady state, obtaining:

$$\begin{bmatrix} \widehat{k}_t \\ \widehat{l}_{t+1} \end{bmatrix} = [J] \begin{bmatrix} \widehat{k}_{t-1} \\ \widehat{l}_t \end{bmatrix} \tag{20}$$

where hat-variables denote percentage deviation rates from their steady-state values and J is the Jacobian matrix of the system (15) and (16) evaluated at the steady state. Its trace, T, and determinant, D, are given by:

$$T = 1 + \frac{1 - \theta(1-s)(1+\chi)(1-a_L) b_L}{(1-s)(1+\chi)(1-a_L) b_L} \tag{21}$$

$$D = \frac{1 - \theta + \theta s (1-a_K) b_K}{(1-s)(1+\chi)(1-a_L) b_L} \tag{22}$$

where, for $i = L, K$, $a_i \equiv a_i(y_*) = \phi_i \frac{\tau_i}{1-\tau_i} > 0$ and $b_i \equiv b_i(y_*) = \frac{1-\tau_i}{1-\tau_i-\tau_Y} > 1$, see (7)-(10), with τ_y, τ_L and τ_K denoting, respectively, the tax rate on income, and the specific tax rates on labour and capital income, all evaluated at the steady state under analysis y_*, i.e., $\tau_y \equiv \frac{\overline{G}}{y_*}$, $\tau_L \equiv \mu_L y_*^{\phi_L}$ and $\tau_K \equiv \mu_K y_*^{\phi_K}$. See (4)-(5).

The local stability properties of the model are determined by the eigenvalues of the Jacobian matrix J or, equivalently, by its trace, T, and determinant, D, which correspond respectively to the product and sum of the two roots (eigenvalues) of the associated characteristic polynomial $Q(\lambda) \equiv \lambda^2 - \lambda T + D$.

In what follows we consider the following assumptions satisfied at the steady state under analysis. As typically done in Woodford economies, we assume that $0 < \theta(1-s) < s < 1/2$, i.e., that the period is short so that θ is small, and that s is also small. Moreover, in accordance with empirical studies and following Lloyd-Braga, Modesto and Seegmuller (2011), we assume that after-tax gross real capital income, $[1-\delta+\rho_t(1-\tau_k(y_t)-\tau_y(y_t))]k_{t-1}$, is increasing with capital and that the after-tax real wage bill, $(1-\tau_L(y_{t+1})-\tau_y(y_{t+1}))\omega_{t+1}l_{t+1}$, is increasing in labor. These two assumptions imply respectively that $1-\theta s(1-b_K)>\theta b_K a_K$ and $(1-a_L)>0$.

All these assumptions are summarized below in Assumption 1 and we consider them satisfied in the rest of the paper.

Assumption 1

1. $0<s<1/2$ *and* $0<\theta<s/(1-s)$
2. $a_K<\frac{1-\theta s(1-b_K)}{\theta b_K}$
3. $a_L<1$

3.1 Analytical Results

Analytical results are easier to obtain with the support of Figure 1, where we have represented in the plane (T,D) three lines relevant for our purpose: the *line* AC $(D=T-1)$ where a local eigenvalue is equal to 1; the *line* AB $(D=-T-1)$, where one eigenvalue is equal to -1; and the *segment* BC $(D=1$ and $|T|<2)$ where two eigenvalues are complex conjugates of modulus 1. When T and D fall in in the interior of triangle ABC the steady state is a *sink (* both eigenvalues with modulus lower than one), i.e., asymptotically stable. In this case, given the present context where only capital is a predetermined variable, the steady state is locally indeterminate[14] and, as known, there are infinitely many stochastic endogenous fluctuations (sunspots) arbitrarily close to the steady state. In all other cases the steady state is locally determinate. It exhibits *saddle* path stability (one eigenvalue with modulus higher than one and one eigenvalue with modulus lower than one) when $|T|>|D+1|$ and it is an unstable *source* (both eigenvalues with modulus higher than one) in the remaining regions.

Straightforward computations show that, under Assumption 1, we always have $D>0$ and $D>-T-1$. Therefore only the 3 shaded regions depicted in Figure 1 are possible. We will have a source when $D>\max\{1,T-1\}$, a saddle when $D<T-1$ and a sink when $T-1<D<1$. Note that if, by continuously changing a parameter of the model, the values of T and D cross the segment BC, a Hopf bifurcation generically occurs (a pair of complex conjugate eigenvalues cross the unit circle). In this case there are

[14] Indeterminacy occurs when the number of eigenvalues strictly lower than one in absolute value is larger than the number of predetermined variables.

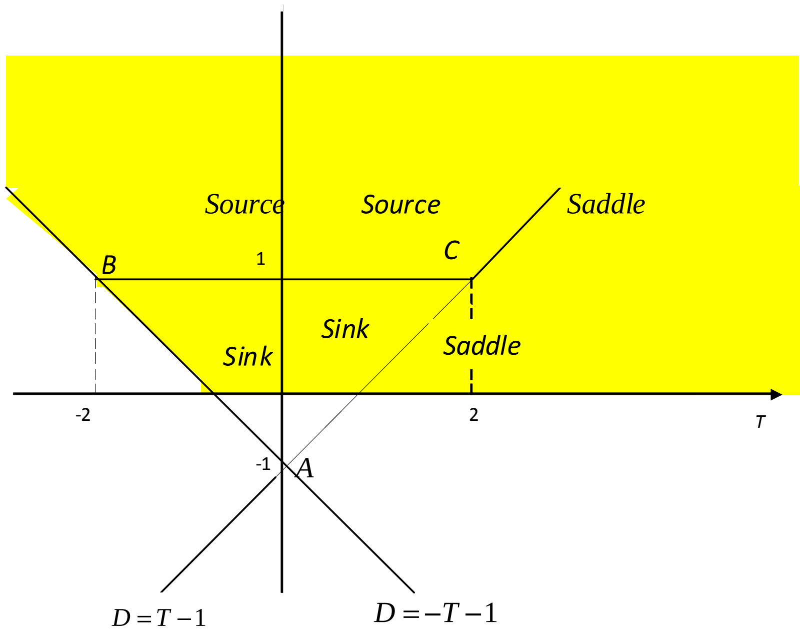

Figure 1: Admissable regions

deterministic cycles describing orbits that lie over an invariant closed curve, surrounding the steady state, in the state space. If the Hopf bifurcation is subcritical this curve emerges when the steady state is a sink and sunspot fluctuations arbitrarily close to the steady state emerge. When the Hopf bifurcation is supercritical the invariant closed curve appears when the steady state is a source and, although sunspot equilibria that stay arbitrarily close to the steady state do not exist, there are nevertheless infinitely many equilibria exhibiting bounded stochastic fluctuations around the invariant closed curve. See for instance Grandmont et al (1998). Also, if, by continuously changing a parameter of the model, the values of T and D cross the line AC, a transcritical bifurcation generically occurs (one eigenvalue crossing the value 1).[15] In this case, if (T, D) is close enough to line AC, two close steady states co-exist. These two steady states exchange stability properties as (T, D) crosses line AC. When (T, D) is on line AC the two steady states collapse into one.

Using (21)-(22) we obtain $D < T - 1 \Leftrightarrow a_K > a_{K2}$, and $D < 1 \Leftrightarrow a_K > a_{K1}$, with a_{K1} and a_{K2} given in the following Proposition:

[15] Pitcfork and Saddle

Proposition 1 *Let Assumption 1 be satisfied and define*

$$a_K^H \equiv \frac{(1-\theta+\theta s b_K)-(1-s)(1+\chi)(1-a_L)b_L}{\theta s b_K} \quad (23)$$

$$a_K^T \equiv \frac{(1-s)(1+\chi)(1-a_L)b_L-(1-s b_K)}{s b_K} \quad (24)$$

Then we have the following:

- *The steady state is a source (unstable) if and only if* $a_K < \min\{a_K^H, a_K^T\}$.
- *The steady state is a saddle if and only if* $a_K > a_K^T$.
- *The steady state is a sink (indeterminate) if and only if* $a_K^H < a_K < a_K^T$,

  where a_L, a_K, b_L *and* b_K *are given by (7)-(10) and evaluated at the steady state under analysis.*

Note that for parameter's values such that $a_K < a_K^T$, if by continuously changing the value of one parameter, a_K crosses the value a_K^H (with (T, D) crossing the interior of segment BC), a Hopf bifurcation may occur. Also, if a_K crosses the value a_K^T, a transcritical bifurcation may occur.

From Proposition 1 we can immediately see that indeterminacy is only possible if $a_K^H < a_K^T$. Accordingly we have the following Corollary:

Corollary 1 *Under Proposition 1 a necessary condition for the occurrence of indeterminacy is that:*

$$(1+\theta)(1-s)(1+\chi)(1-a_L)b_L > 1 \quad (25)$$

A sufficient condition for (25) is that the labor market exhibits the "wrong slopes" condition, i.e., a downward sloping labor supply steeper than the labor demand curve. Although at the individual level labor supply in infinitely elastic, at the general equilibrium level this elasticity becomes $(1+\chi)(1-a_L b_L)/[1-(1+\chi)(1-a_L b_L-(1-b_L)(1-s)]$.[16] Since the elasticity of the labor demand curve is $-s$, the "wrong slopes" condition $(1-s)(1+\chi)(1-a_L)b_L > 1$ is sufficient for (25). So, once more the emergence of indeterminacy is related with the slopes of the labor demand and supply schedules.[17]

[16] At the general equilibrium level we should take into account that both the labor income tax and average consumption are functions of labor income.

[17] See for example Benhabib and Farmer (1994), Barinci and Chéron (2001) and Dufourt et al. (2008).

Note that when tax rates on capital and labor income are constant, i.e. tax rate cyclicality is absent ($\phi_K = \phi_L = 0$ implying $a_K = a_L = 0$) indeterminacy requires either $\chi > 0$ and/or $b_L > 1$, since otherwise condition (25) above cannot be satisfied under Assumption 1. As we can only have $b_L > 1$ when $\tau_Y \equiv \frac{\overline{G}}{y_*} > 0$, see (9), consumption externalities and/or structural government spending are necessary for indeterminacy in the absence of cyclicality of tax rates. Accordingly, we have:

Corollary 2 *Under Proposition 1, consumption externalities, $\chi > 0$, and/or structural government spending, $\bar{G} > 0$, are necessary for indeterminacy in the absence of cyclicality of tax rates on capital and labor income ($\phi_K = \phi_L = 0$).*

Corollary 3 below establishes the necessary and sufficient condition for the emergence of indeterminacy in the absence of cyclical tax rates. From Proposition 1 we can see that in this case we must have $a_K^H < 0 < a_K^T$. Note that $a_K^H < 0 \Leftrightarrow (1-s)(1+\chi) b_L > (1-\theta+\theta s b_K)$, which under Assumption 1 implies that $a_K^T > 0$ is also satisfied, i.e. we have that $(1-s)(1+\chi) b_L > (1-sb_K)$.[18] Hence we obtain:

Corollary 3 *Under Assumption 1 and from Proposition 1, indeterminacy occurs in the absence of cyclical capital and labor income tax rates ($\phi_K = \phi_L = 0$) if and only if $(1-s)(1+\chi) b_L > (1-\theta+\theta s b_K)$, implying also that $(1-s)(1+\chi) b_L > (1-sb_K)$.*

As explained above bounded endogenous fluctuations (caused by autonomous changes in expectations even when fundamentals are stationary) may emerge around the steady state considered when the later is a sink (indeterminate) or even a source (supercritical Hopf). Also, when the steady state is locally saddle stable there are no endogenous fluctuations arbitrarily **close to it**. However this does not guarantee that larger endogenous fluctuations do not exist. In Section 4, departing from a situation where local indeterminacy (and thereby local cycles) would exist in the absence of cyclical specific tax rates on capital and labor, we show that by introducing sufficient procyclical specific tax rates the goverment is able to ensure local saddle path stability. However, in Section 5 we also show that under procyclical specific tax rates the steady state is not unique and another steady state where output is lower and which is a source or a sink, exists. This means

[18] Indeed, $(1-\theta+\theta s b_K) > (1-sb_K)$. Since $b_K > 1$ ((see ()), we have $1-\theta+\theta s b_K > 1-\theta+\theta s$ and $1-s > 1-sb_K$. Since under Assumption $\theta(1-s) < s$, we have that $1-\theta+\theta s > 1-s$ and therefore $(1-\theta+\theta s b_K) > (1-sb_K)$.

that with sufficiently procyclical tax rates, local instability associated with small volatility of expectations is eliminated, but the risk of larger fluctuations, linked to larger autonomous changes in expectations, leading to lower values of output, still exists. We conclude that tax rate procyclicality is not able to guarantee global stabilization with respect to endogenous cycles and autonomous self fulfilling volatile expectations.

4 Local Stabilization Policy

In this Section we discuss the role of cyclical tax rates on local stabilization. For the sake **of exposition** and simplicity we consider the normalized state $y_* = y_{*N} \equiv 1$ where, using (7)-(10) the following applies:

$$\begin{aligned} \tau_L &= \mu_L, \tau_K = \mu_K, \text{and } \tau_y = \overline{G} \\ a_L &= \phi_L \frac{\mu_L}{1-\mu_L}, \quad a_K = \phi_K \frac{\mu_K}{1-\mu_K} \\ b_L &= \frac{1-\mu_L}{1-\mu_L-\overline{G}}, \quad b_K = \frac{1-\mu_K}{1-\mu_K-\overline{G}} \end{aligned} \tag{26}$$

Using Definition 2, we obtain the following Proposition ensuring existence of the normalized steady state.

Proposition 2 ***Normalized Steady State:***
Define

$$\begin{aligned} l_{*N} &= \left[(1-s)\left(1-\mu_L-\overline{G}\right)\right]^{1+\chi}\left(\overline{G}^{\eta}/B\right) \\ k_{*N} &= \left(\left[(1-s)\left(1-\mu_L-\overline{G}\right)\right]^{1+\chi}\left(\overline{G}^{\eta}/B\right)\right)^{-(1-s)/s}, \text{ and} \\ y_{*N} &\equiv 1 \end{aligned}$$

Then $(k_*, l_*) = (k_{*N}, l_{*N})$ with the corresponding level of output $y_* = y_{*N}$ is a steady state of the dynamic system (15)-(16) if and only if

$$B = B_* \equiv \left[1-\mu_K-\overline{G}\right]\left[(1-s)\left(1-\mu_L-\overline{G}\right)\right]^{1+\chi\frac{1-s}{s}}(1-s)^{1+\chi}\,\overline{G}^{\eta}\left[\frac{\beta s}{\theta}\right]^{\frac{s}{(1-s)}}.$$

Moreover, at the normalized steady state y_{*N}, the tax rates τ_L, τ_K, τ_y and a_L, a_K, b_L and b_K are given by (26).

To illustrate the discussion we have depicted in Figure 2, in the plane (a_L, a_K), the functions a_{K2} and a_{K1} given in Proposition 1, that define the sink, saddle and source regions for value of the parameters empirically plausible and consistent with Assumption 1 and Corollary 3. Indeed, in order to discuss how tax policy can be used to ensure local saddle path stability, eliminating (local) expectations driven cycles, we considered parameter values consistent with the emergence of indeterminacy in the absence of cyclicality of tax rates in the neighbourhood of the normalized steady state.[19] We considered $\beta = 0.99$ and $\delta = 0.025$, in line with most calibrations used in the business cycle literature for quarterly data. Hence, $\theta = 0.03475$, and we fix $s = 0.35$, so that $\theta(1-s) < s < 0.5$, as required by Assumption 1. Concerning tax parameters we set $\tau_y(= \overline{G}) = 0.23$, $\mu_L = 0.20$ and $\mu_K = 0.06$, implying a total labor income tax rate of 0.43 and a total capital income tax rate of 0.29. These two last figures are in line with the ones reported in Mendonza et al. (1994) for European countries and are also consistent with reported ratios of tax revenues in GDP around 40% for the euro area in 2011.[20] To fix the value of $\overline{G}$ we considered that only 45% of government spending was non wasteful. Using again values for Europe, where on average government spending represents 51% of GDP,[21] we arrived at a value of 0.23.[22] Finally, we fix $\chi = 0.1$, a value consistent with the empirical findings of Maurer and Meier (2008).

The first remark is that for sufficiently negative values of a_K and a_L the steady state is a sink, when a_L is sufficiently positive and a_K is sufficiently negative the steady state is a source and for sufficiently positive values of a_K and a_L the steady state is a saddle. Note that we consider different possible values for a_K and a_L, for fixed values of μ_K and μ_L, while ϕ_K and ϕ_L vary.[23] Since at the normalized steady state (26) applies, the steady is a saddle for sufficiently positive values of ϕ_K and ϕ_L, i.e., for sufficiently procyclical tax rate rules. The steady state is a sink for sufficiently negative values of ϕ_K and ϕ_L, i.e., for sufficiently countercyclical tax rate rules. Moreover when ϕ_K crosses the value $\phi_K^T \equiv a_{K2}\frac{1-\mu_K}{\mu_K}$ a transcritical bifuraction occurs. Also, when $a_K > a_{K1}$ and ϕ_K crosses $\phi_K^H \equiv a_{K1}\frac{1-\mu_K}{\mu_K}$ a Hopf bifurcation occurs

[19] This implies that in Figure 2 for $a_L = a_K = 0$ we have $a_{K1} < 0$ and $a_{K2} > 0$. See Proposition 1.

[20] See Eurostat: Statistic in focus 55/2012. Indeed we have that $0.43(1-s) + 0.29s = 0.381$.

[21] See Eurostat: Statistic in focus 33/2012.

[22] This figure includes health (7.5%) and education (5.5%) expenditures, defence, public order and safety, environmental protection, housing and culture (6.6%) and 50% of general public services (3,4%). It excludes social pretection expenditures (20%) since in this model we not address redistribution policies.

[23] Note that ϕ_K and ϕ_L do not affect the values of k, l and y at the normalized steady state. See Corollary 2.

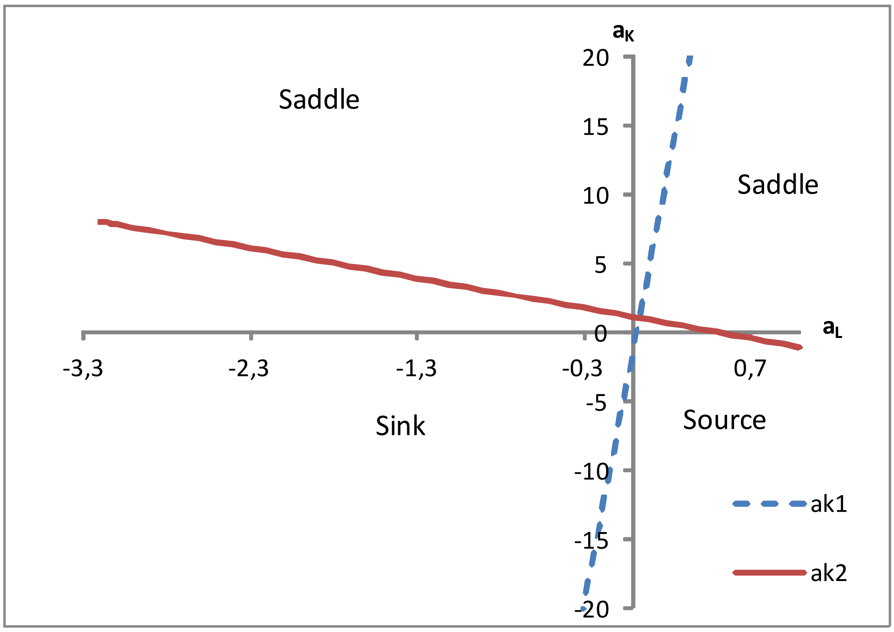

Figure 2: Saddle, sink and source regions

To ease the presentation we start by discussing the stabilization effects of cyclical labor and capital separately. We assume first that the government can only use cyclical labor income tax rates and then we analyse the case where cyclical capital income tax rates are the only stabilization tool

4.1 Cyclical labor income tax rates

In this section, cyclical labor income taxation is the only stabilization tool considered, so that we have $\phi_K = 0$ and $a_K = 0$. We start by characterizing the local stability properties of the normalized steady state in terms of a_L when cyclicality of the capital income tax rate is absent. From Proposition 1, after some simple calculations, the following applies:

Corollary 4 *Under Assumption 1, let $b_K < 1/s$, and define a_L^T and a_L^H as*

$$a_L^T \equiv 1 - \frac{(1 - sb_K)}{(1 - s)(1 + \chi)b_L} \tag{27}$$

$$a_L^H \equiv 1 - \frac{(1 - \theta + \theta sb_K)}{(1 - s)(1 + \chi)b_L} \tag{28}$$

where $1 > a_L^* > a_L^{**}$,[24] *with* a_L, b_L *and* b_K *given by (26). Then, from Proposition 1, for* $\phi_K = 0$ *so that* $a_K = 0$, *we have that:*

- *The normalized steady state is a source (unstable) if and only if* $a_L^H < a_L < a_L^T$;
- *The normalized steady state is a saddle if and only if* $1 > a_L > a_L^T$;
- *The normalized steady state is a sink (indeterminate) if and only if* $a_L < a_L^H$.

Note that when a_L crosses the critical value a_L^T a transcritical bifurcation may occur, whereas when it crosses the critical value a_L^H a Hopf bifurcation may occur.

From Proposition 4, the condition which guarantees that the normalized steady state is a saddle is $a_L > a_L^T$, with a_L^T given in (27). Under Corollary 3 we have $a_L^T > 0$. Hence labor income tax rates should be sufficiently procyclical to garantee the emergence of a saddle, i.e. we must have $\phi_L > \phi_L^T \equiv a_L^T \frac{1-\mu_L}{\mu_L}$. We have the following result:

Proposition 3 *Let Assumption 1 and the conditions of Proposition 2 and Corollary 3 be verified, and further assume that* $b_K < 1/s$ *at the normalized steady state. Then, under Corollary 4, in the absence of cyclical capital income tax rates,* $\phi_K = a_K = 0$, *a sufficiently procyclical labor income tax rate,* $\phi_L > \phi_L^T \equiv a_L^T \frac{1-\mu_L}{\mu_L}$, *with* a_L^T *given in (27), is able to guarantee local saddle path stability of the normalized steady state.*[25]

Note however that when $\phi_L > \phi_L^T$ the total tax rate faced by workers, $\tau_L(y) + \tau_y(y)$, will only be procyclical, i.e., increasing in y, if $\phi_L \tau_L(y) > \tau_y(y)$. For example, under our calibration, $a_L > a_L^T$ becomes $a_L > 0.465$ so that, since $\mu_L = 0.20$, using (7), the government, by choosing $\phi_L > 1.86$, guarantees local saddle path stability. Since in our calibration $\overline{G} = 0.23$, when $\phi_L > 1.86$ we have that $\phi_L \mu_L > \overline{G}$, and therefore, around the normalized steady state, the total tax rate faced by workers is procyclical.

To understand why a sufficiently procyclical labor income tax rate eliminates local indeterminacy and cycles driven by self-fulfilling volatile expectations, consider that at some period t, departing from a steady state equilibrium, agents expect an increase in future output. This leads to a decrease

[24] In footnote XXX we have shown that under Assumption 1 the following obains: $(1 - \theta + \theta s b_K) > (1 - \theta + \theta s b_K)$. Hence, $a_L^T > a_L^H$. The condition $b_K < 1/s$ ensures that $a_L^T < 1$.

[25] Of course, since under Assumption 1 $a_L < 1$, ϕ_L cannot be too high, i.e., $\phi_L < \frac{1-\mu_L}{\mu_L}$.

in $\tau_y(y_{t+1})$. However, with a sufficient procyclical labor income tax rate $\tau_L(y_{t+1})$, the increase in expected output is likely to end up implying an increase in expected future total labor income tax rate, $\tau_y(y_{t+1}) + \tau_L(y_{t+1})$, leading to a decrease in current labor supply. See (15). Hence, the current marginal productivity of capital (the real interest rate), ρ_t, decreases (see (2)) and so does capital accumulation. See (16). This implies that future output tends to decrease, which contradicts the initial expectation. Therefore the initial change in expectations is not fulfilled so that (local) fluctuations driven by self-fulfilling volatile expectations are not possible.

4.2 Using only cyclical capital income tax rates to stabilize the economy

In this section capital income taxation is the only stabilization instrument considered.

When we only have cyclical capital income tax rates, $\phi_L = 0$ so that we have $a_L = 0$. As in the labor taxation case, the government can eliminate fluctuations locally, guaranteeing the existence of a saddle even if capital income taxation is the only available instrument. In this case, using Proposition 1, the normalized steady state is a saddle when:

$$a_K > a_K^T(\phi_L = 0) \equiv \frac{(1-s)(1+\chi)\, b_L - (1 - sb_K)}{sb_K} \tag{29}$$

with a_K, b_L and b_K given by (26).

Note that under Corollary 3 we have $a_K^T(\phi_L = 0) > 0$. Hence, capital income taxes should be sufficiently procyclical to stabilize the economy, $\phi_K > \phi_K^T(\phi_L = 0) \equiv a_K^T(\phi_L = 0)\frac{1-\mu_K}{\mu_K}$. However, from Assumption 1 we must have $a_K^T(\phi_L = 0) < \frac{1-\theta s(1-b_K)}{\theta b_K}$, so that the set of procyclical policies that stabilize the economy is not empty. The condition $a_K^T(\phi_L = 0) < \frac{1-\theta s(1-b_K)}{\theta b_K}$ is equivalent to $\theta(1-s)\left[(1+\chi)\, b_L - 1 - s(1-sb_K)\right] < s$, a condition verified under typical calibrations where θ is rather small. Accordingly we have the following Proposition:

Proposition 4 *Assume that $\theta(1-s)\left[(1+\chi)\, b_L - 1 - s(1-sb_K)\right] < s$. Then, under Corollary 3, without cyclical labor income tax rates, $\phi_L = a_L = 0$, a sufficiently procyclical capital income tax rate, $\phi_K > \phi_K^T(\phi_L = 0) \equiv a_K^T(\phi_L = 0)\frac{1-\mu_K}{\mu_K}$ with $a_K^T(\phi_L = 0)$ given in (29) is able to guarantee saddle path stability of the normalized steady state.*

As before, sufficiently procyclical capital income tax rates stabilize the economy. This result is novel and rehabilitates the role of capital income taxation as a stabilization tool. Note however that when $\phi_K > \phi_K^T(\phi_L = 0)$ the total tax rate faced by capitalists, $\tau_K(y) + \tau_y(y)$, will only be procyclical if $\phi_K \tau_K(y) > \tau_y(y)$. To illustrate these results we use again our calibration. In this case, as $a_K^T(\phi_L = 0) = 1.01$, the government, by choosing a $\phi_K > 15.8$ guarantees the emergence of a saddle preventing local belief driven cycles. Also as $\overline{G} = 0.23$ and $\mu_K = 0.06$, when $\phi_K > 15.8$ we have that $\phi_K \mu_K > \overline{G}$, and therefore, around the normalized steady state, the total tax rate faced by capitalists is procyclical.

To understand why a sufficiently procyclical capital income tax rate eliminates local indeterminacy and sunspots driven by self-fulfilling volatile expectations, consider that at period t, departing from a steady state equilibrium, agents expect an increase in future output. Therefore, the expected future tax rate on income $\tau_{y_{t+1}}$ decreases (see (4)). This implies an increase in current labor supply (see (15)) which leads to an increase in the current marginal productivity of capital (the real interest rate), ρ (see (2)), that by itself would increase capital accumulation, helping the initial change on expectations. See (16). However, the increase in current labor supply implies an increase in current output and with a procyclical tax rate on capital income capital accumulation tends to decrease implying that future output tends to decrease. If the tax rate is sufficiently procyclical this last chanel dominates so that the initial change in expectations is not fulfilled and (local) cycles driven by self-fulfilling volatile expectations do not emerge.

4.3 Using both cyclical labor and capital income tax rates to stabilize the economy

In this case, from Proposition 1, the condition that garantees saddle path stability $a_K > a_{K2}$ can be written **at the normalized steady state** as:

$$(1-s)(1+\chi)b_L a_L + s b_K a_K > (1-s)\,(1+\chi)\,b_L - (1 - s b_K). \tag{30}$$

with a_L, a_K, b_L and b_K given by (26).Note first that, if there is indeterminacy without cyclicality of specific tax rates, then the RHS of (30) is positive. See Corollary 3. Therefore, sufficiently positive values of a_L and a_K guarantee the emergence of a saddle.[26] Accordingly we have the following Proposition:

Proposition 5 *Let Assumption 1 be verified and further assume that $sb_K < 1$ and $\theta(1-s)\left[(1+\chi)\,b_L - 1 - s(1 - sb_K)\right] < s$ at the normalized steady state.*

[26]Note that under Assumption 1, the LHS of (30) has an upper bound. However, this upper bound is higher than the RHS if, as before in Propositions 4 and **??** , we assume that $\theta(1-s)\left[(1+\chi)\,b_L - 1 - s(1 - sb_K)\right] < s$ and that $b_K < 1/s$.

Then, under Corollary 3, sufficiently procyclical tax rates on capital and labor income ensure local saddle path stability for the normalized steady state and are able to eliminate local indeterminacy caused by positive consumption externalities, and structural government spending.

Since $(1-s)(1+\chi)b_L - (1-sb_K) > 0$ at the normalized steady state, from (30) we can see that it is a priori possible to ensure local saddle path stability even if the specific tax rate on capital income is acyclical or countercyclical, i.e., $a_K \leq 0$ and $\phi_K \leq 0$, provided a_L and ϕ_L are sufficiently positive, i.e., provided the specific tax rate on labor income is sufficiently procyclical. This would require $a_L > \frac{(1-s)(1+\chi)b_L-(1-sb_K)-sb_Ka_K}{(1-s)(1+\chi)b_L}$. However, since under Assumption 1 $a_L < 1$, this will not be possible if a_K and ϕ_K are too negative. More precisely, to garantee that $a_L < 1$ we must have $\frac{(1-sb_K)+sb_Ka_K}{(1-s)(1+\chi)b_L} > 0$ or equivalently $0 \geq a_K > -\frac{1-sb_K}{sb_K}$. It is also possible to ensure local saddle path stability even if the specific tax rate on labor income is acyclical or countercyclical, i.e., $a_L \leq 0$ and $\phi_L \leq 0$, provided a_K and ϕ_K are sufficiently positive, i.e., provided the specific tax rate on capital income is sufficiently procyclical. This would require $a_K > \frac{(1-s)(1+\chi)b_L(1-a_L)-(1-sb_K)}{sb_K}$. However, since $a_K < \frac{1-\theta s(1-b_K)}{\theta b_K}$ under Assumption 1, this is only possible for $0 \geq a_L > \frac{\theta\left[(1-s)(1+\chi)b_L-(1-sb_K)-s^2(b_K-1)\right]-s}{\theta(1-s)(1+\chi)b_L}$. Accordingly we have the following Proposition.

Proposition 6 *Let Assumption 1 be verified and assume further that that* $sb_K < 1$ *and* $\theta(1-s)\left[(1+\chi)b_L - 1 - s(1-sb_K)\right] < s$ *at the normalized steady state. Then, under Corollary 3, we have the following:*

The higher the degree of procyclicality of the capital income tax rate, the lower the degree of procyclicality of the labor income tax rate required to guarantee saddle path stability of the normalized steady state. If the capital income tax is sufficiently procyclical, saddle path stability can even be obtained with a constant or countercyclical tax rate on labor income such that $\frac{\theta(1-s)[(1+\chi)b_L-1-s(1-sb_K)]-s}{\theta(1-s)(1+\chi)b_L} < a_L \leq 0$.

The higher the degree of procyclicality of the labor income tax rate, the lower the degree of procyclicality of the capital income tax rate required to guarantee saddle path stability of the normalized steady state. If the labor income tax rate is sufficiently procyclical, saddle path stability of the normalized steady state can even be obtained with a constant or countercyclical tax rate on capital income such that $-\frac{1-sb_K}{sb_K} < a_K \leq 0$.

This last result, implies that labor and capital taxation can be seen as local substitutable stabilization tools. Therefore, in order to stabilize locally the economy, governments can choose different combinations of procyclical and countercyclical labor and capital tax rates. This is a new result and validates the current policy debate on how the tax burden should be divided between labor and capital income.[27]

5 Steady State Uniqueness/Multiplicity and Stability

From Definition 1, a steady state solution satisfies $H(y) = \overline{H} > 0$ with $z_K(y) \equiv 1 - \tau_K(y) - \tau_y(y) > 0$ and $z_L(y) \equiv 1 - \tau_L(y) - \tau_y(y) > 0$. In order to study the existence of multiple steady states, it will be important to analyse whether $H(y)$ is a monotonous function. Therefore, let us compute $H'(y)$, the first derivative of $H(y)$:

$$\begin{aligned} H'(y)\frac{y}{H(y)} &= \frac{1-s}{s}\chi - (1+\chi)\frac{1-s}{s}\frac{\tau_L(y)\phi_L}{z_L(y)} - \frac{\tau_K(y)\phi_K}{z_K(y)} + \\ &+\tau_y(y)\left((1+\chi)\frac{1-s}{z_L(y)} + \frac{1}{z_K(y)}\right) \end{aligned} \tag{31}$$

Note that from (9)-(10) we have $\frac{\tau_y(y)}{1-\tau_i(y)-\tau_y(y)} = b_i(y) - 1$ so that, using (7)-(8) and rearranging terms, we can rewrite (31) as:

$$H'(y)\frac{y}{H(y)} = b_K(y)\left[\frac{(1-s)(1+\chi)(1-a_L)b_L(y) - (1 - sb_K(y))}{sb_K(y)} - a_K(y)\right] \tag{32}$$

The first term within brackets in (32), when evaluated at a steady state solution y_*, corresponds to a_K^T as given in Proposition 1. Hence, using (32) and Proposition 1 we can immediately see that a steady state solution y_* is a saddle if and only if $H'(y)\frac{y}{H(y)} < 0$ at y_*. Accordingly we have the following Proposition:

Proposition 7 *Under Assumption , a steady state solution y_* is:*

- *a saddle if and only if $H(y)$ is decreasing at y_*, i.e, if $H'(y_*) < 0$, where $a_K > a_K^T$;*

[27] Guo (1999) found that, in a one-sector RBC model with strong increasing returns in production, progressive labor income taxation can stabilize the economy against sunspot fluctuations, when the capital tax schedule is flat, i.e., $a_K = \phi_K = 0$. Proposition 6, taking into account that progressivity tends to generate similar results in terms of local stability properties as procyclicality, extends somehow that result for the case of $\phi_K < 0$.

- *a source or a sink when $H(y)$ is increasing at y_*, i.e, if $H'(y_*) > 0$, where $a_K < a_K^T$,*

  with a_K^T given in (24) and a_K given by (8)-(10) all evaluated at the steady state under analysis.

We consider that Proposition 2 is satisfied so that the normalized steady state $y_{*N} = 1$ always exists. We analyse now if this normalized steady state is unique or not. We show below that with countercyclical tax rates the normalized steady state $y_{*N} = 1$ is unique, being a source or indeterminate.We then consider a procyclical tax rate on labor income and a constant tax rate on capital income, under which y_{*N} is a saddle (i.e., $a_L > a_L^T$, with a_L given by (26) and a_L^T given in 4). Under these conditions we show that there is at least another steady state with a lower level of output, $y_{*a} < 1$, that it is a sink or a source. A similar result applies when a procyclical tax rate on capital income (with a constant tax rate on labor income) is considered instead.

5.1 Countercyclical Tax rates

Here we discuss the case of countercyclical tax rates, i.e., $\phi_K < 0$ and $\phi_L < 0$. In the Appendix we show that a steady state y_*, if it exists, is unique and such that $H'(y_*) > 0$. Accordingly we have:

Proposition 8 *Under Proposition 2, in the presence of countercyclical tax rates on labour and/or capital income, the normalized steady state $y_{*N} \equiv 1$ is unique with $H'(1) > 0$.*

However, this steady state never exhibits saddle path stability. Indeed, combining the last two Propositions above, we conclude that with countercyclical tax rates the unique steady state is either a sink or a source. In the first case, local indeterminacy and sunspots emerge. Moreover, under Proposition 1 and Proposition 2, when a_K (evaluated at y_{*N}) decreases, as ϕ_K is made to vary and decrease continuously, crossing the value $\phi_K^H \equiv a_K^H \frac{1-\mu_K}{\mu_K}$ a Hopf bifurcation occurs, and the steady becomes a source for $a_K < a_K^H$. If the Hopf bifurcation is supercritical an invariant closed curve appears in the state space, surrounding the source steady state. In this case, deterministic and stochastic endogenous fluctuations bounded by the invariant closed curve emerge, even with a determinate (source) steady state. Therefore, as also emphasized in Guo and Lansing (2002), fiscal policies able to eliminate local indeterminacy may not succeed in stabilizing the economy with respect to volatile self fulfilling expectations.

5.2 Procyclical labor income tax rates

In this section we consider procyclical tax rates on labor income, i.e., $\phi_L > 0$, and we assume that cyclicality of the tax rate on capital income is absent, i.e. $\phi_K = 0$, as in Section 4.1. We further assume that a normalized steady state $y_* = y_{*N} \equiv 1$ exists, satisfying the conditions of Proposition 2. In the Appendix we show that, under these conditions, there are generically an even number of steady states, y_{*N} and at least another co existing steady state y_{*A}, except when a_L crosses the value a_L^T, with a_L given in (7) and a_L^T given in (4), both evaluated at y_{*N}. In this last situation the normalized steady state undergoes a transcritical bifurcation: for low values of a_L, with $a_L < a_L^T$, the normalized steady state is a sink or a source and there is another steady state with higher values of output $y_{*A} > y_{*N}$ which is a saddle. When $a_L = a_L^T$ the two steady states coincide, while as a_L increases further the normalized steady state, y_{*N}, becomes a saddle and the other steady state, now with lower values of output $y_{*A} < y_{*N}$, is a source or a sink.

Figure 3 below illustrates the transcritical bifurcation and cases where two steady states exist, using the same parameter values as in Figure 2. The horizontal line $Hbar$ represents $\overline{H}$ and the curve H represents the function $H(y)$ for the values of ϕ_L considered. We obtain the curve H_T for $\phi_L = 1.8611$ (this value being identical to $a_L^T \frac{1-\mu_L}{\mu_L} = 1.861$, so that $a_L = a_L^T$) where the normalized seady state (under conditions of Proposition 2) is the unique steady state. The curve H_1 is obtained for $\phi_L = 1.1$ (so that $a_L < a_L^T$). Finally the curve H_2 is obtained for $\phi_L = 3.5$. We can see that in this case $H'(y_{*N} = 1) < 0$ and, therefore, the normalized steady state is a saddle (see also Proposition 7) satisfying the conditions of Proposition 3. However, another steady state, y_{*A}, with a lower level of output $y_{*A} < y_{*N}$, coexists and, since $H'(y_{*A}) > 0$, it can be a source or a sink and can even undergo a Hopf bifurcation (see Proposition 1 and 7). Therefore even if a sufficiently procyclical tax rate is able to ensure local saddle path stability it is not able to eliminate, a priori, the possibility of larger flcutuations around a steady state with a lower level of output (through heteroclinic bifurcations for instance).

Accordingly, we have the following Proposition.

Proposition 9 *Under Proposition 2 and Proposition 3, consider the existence of a normalized steady state y_{*N} with $\phi_L > \phi_L^T$, so that y_{*N} is locally a saddle. Then, there is another steady state with a lower level of output, which is a source or a sink.*

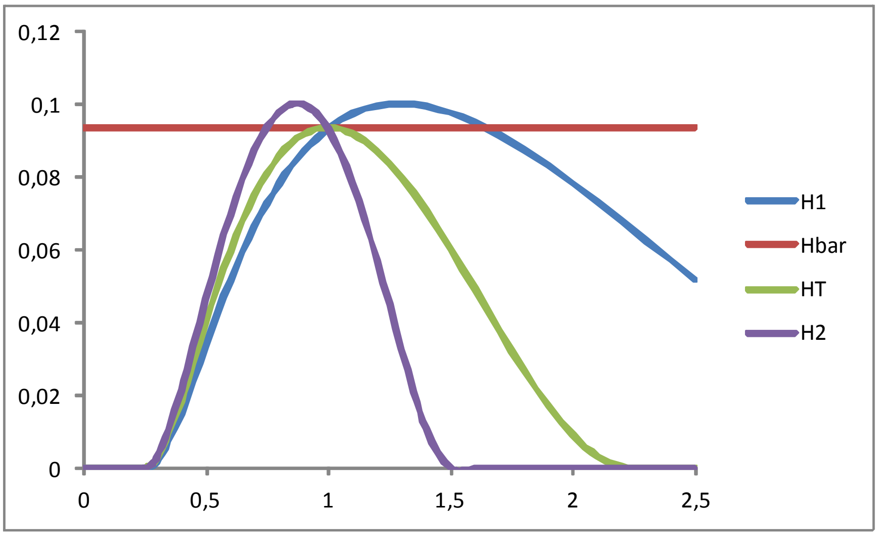

Figure 3: Steady State Multiplicity: the transcritical bifurcation

6 Discussion of the results

We have seen that countercyclical specific tax rates are not able to eliminate local indeterminacy and sunspots caused by consumption externalities and constant structural public expenditures. In contrast, sufficiently procyclical labor and capital income specific tax rates (and therefore procyclical government spending) are able to stabilize locally an economy with consumption externalities and constant structural public expenditures, eliminating business cycles driven by self-fulfiling prophecies that stay arbitrarily close the steady state. These findings confirm previous insights about the local stabilization effects of procyclical specific labor income taxation. However the result that procyclical capital taxes can also eliminate local expectations driven fluctuations is new. Our work therefore rehabilitates the role of capital income taxes as a local stabilization tool.

However, as we have shown, procyclical specific tax rates lead to steady state multiplicity, and whenever the steady state under analysis becomes a saddle path there is at least another steady state with a lower level of output that is either a source or indeterminate and Hopf bifurcations may occur. Hence, depending on expectations, the economy may end up converging a lower level of output and it is not completely, or globally, insulated from instability linked to volatile expectations.

7 Concluding Remarks

Our framework of analysis is in accordance with empirical evidence and we believe it to be particularly well suited to study policy choices under the current situation of strained public accounts, observed in many developed economies. Indeed, the desire to 'keep up with the Joneses' is not only supported by empirical studies,[28] but it is also regarded as one of the possible causes behind the increase in consumption and debt that helped to spread the current financial and economic crisis.[29] Empirical studies also confirm that public goods and infrastructures may influence the utility of consumption.[30] Moreover, the Woodford (1986) framework, where workers are finance constrained and save only in the form of money, is closer to the situation existing in most countries, where other assets are held only by a very small fraction of the population.[31] Note that the financial crisis seems to have exacerbated this feature, increasing the strength of credit constraints. Finally, considering separately labor and capital income taxation is consistent with what we observe in most countries, where tax rules for capital and labor income are different. From a policy point of view it is also important to discuss separately the effects of these two types of taxation, specially within the current economic policy agenda, where countries, forced to reach a balanced budget, are discussing which type of income they should tax and how. Moreover our framework also allows us to adress jointly the (de)stabilizing effects of the structural public expenditures, another hot topic in the current policy agenda, and taxation choices.

We conclude by noting that our results also can be used to comment on recent fiscal consolidation policies. In the current economic crisis, countries of the EU periphery were forced to drastically reduce their public deficits and are strongly advised to keep a balanced budget. One possible way is to increase tax rates, but many economic analysts, fearing that this increase in tax rates would reinforce the crisis and create instability, recommended instead a cut in government expenditures and in specific tax rates. Our results show that this type of (procyclical) fiscal policy rule stabilizes the economy with respect to small changes in self fulfilling expectations of future

[28] See for example Carlsson et al. (2003), Ferrer-i-Carbonell (2005) and Mauer and Meier (2008).

[29] See, for instance, Barba and Pivetti (2009).

[30] See for example Amano and Wirjanto (1998), Evans and Karras (1996), Karras (1994) and Ni (1995).

[31] According to Banks et al. (2000) most american and british households have very few financial assets: median financial wealth in both countries is only a few thousand dollars. In Portugal, for the total population, financial assets (60% of which are saving deposits) represent only 12% of net wealth. See INE-ISFF (2010).

economic activity, but not with respect to higher pessimistic changes in expectations. This might explain the increasing concern of policy makers with the management of expectations, in order to prevent the emergence of bad equilibria.[32]

8 Appendix

8.1 Uniqueness of steady state with countercyclical tax rates

In view of Definition 1, steady state solutions y must satisfy (**??**) with $z_K(y) \equiv 1 - \tau_K(y) - \tau_y(y) > 0$ and $z_L(y) \equiv 1 - \tau_L(y) - \tau_y(y) > 0$. Using (5), we have here that $z_K(y) = 1 - \mu_K y^{\phi_K} - \frac{\overline{G}}{y} > 0$ and $z_L(y) = 1 - \mu_L y^{\phi_L} - \frac{\overline{G}}{y} > 0$, where $\phi_K < 0$ and $\phi_L < 0$ in the case of countercyclical tax rates. Then, both functions $z_K(y)$ and $z_L(y)$ are increasing in y. Moreover $lim_{y \to 0} z_L(y) = -\infty$ and $lim_{y \to \infty} z_L(y) = 1$. Hence $z_L(y) > 0$ for sufficiently high values of y, the same happening for $z_K(y)$. Therefore, we have that $z_K(y) > 0$ simultaneously with $z_L(y) > 0$ for sufficiently high values of y. Moreover, $H'(y) > 0$ for all y sufficiently high under which $z_K(y) > 0$ and $z_L(y) > 0$. Indeed, since $z_K(y) > 0$ and $z_L(y) > 0$, the two middle terms on the RHS of (31) are positive when tax rates on labor and capital income are countercyclical, i.e. $\phi_L \leq 0$ and $\phi_K \leq 0$. Since $H(y) > 0$ with $z_K(y) > 0$ and $z_L(y) > 0$ is a continuous function it can only cross once the positive value $\overline{H}$. Accordingly, a steady state y_*, if it exists, must be unique and such that $H'(y_*) > 0$.

8.2 Multiplicity of steady state with procyclical tax rates

We assume that a normalized steady state $y_* = y_{*N} \equiv 1$ exists, satisfying the conditions of Proposition 2, and further consider in this subsection a procyclical tax rate on labor income, i.e., $\phi_L > 0$, and that cyclicality of the tax rate on capital income is absent, i.e. $\phi_K = 0$, as in Section 4.1. Therefore $z_i(y) \equiv 1 - \tau_i(y) - \tau_y(y)$ can be written as $z_K(y) = 1 - \mu_K - \frac{\overline{G}}{y}$ and $z_L(y) = 1 - \mu_L y^{\phi_L} - \frac{\overline{G}}{y}$. As referred before, in view of Definition 1, steady state solutions y must satisfy (**??**) with $z_K(y) > 0$ and $z_L(y) > 0$. The first function $z_K(y)$, which is increasing in y, only takes positive values for $y > y_c \equiv \frac{1-\mu_K}{\overline{G}}$.

[32]Indeed the importance of (self-fulfilling) expectations has been clearly recognized by policy makers of the European periphery, that are determined to change markets expectations and perceptions in order to restore credibility and confidence. See Gaspar (2012).

Therefore, the normalized steady state must satisfy $y_{*N} = 1 > y_c$. On the contrary, computing the derivative $z_L'(y) = \left[\bar{G} - \phi_L \mu_L y^{\phi_L+1}\right] y^{-2}$, we see that $z_L(y)$ is increasing for $y < y_d \equiv \left[\frac{\bar{G}}{\phi_L \mu_L}\right]^{\frac{1}{\phi_L+1}}$ and decreasing for higher values of y. Hence $z_L(y)$ has a maximum at the critical value y_d, given by $z_L(y_d) = 1 - \left(\bar{G}^{\phi_L} \mu_L\right)^{\frac{1}{\phi_L+1}} (\phi_L + 1)\, \phi_L^{-\frac{\phi_L}{\phi_L+1}}$. Of course $z_L(y_d)$ and $z_L(y = 1)$ must be positive under the conditions of Definition 1 and Proposition 2. Since $z_L(y)$ is a continuous function and $z_L(0) = z_L(+\infty) = -\infty$, it must cross the value zero at two critical values, y_a and y_b such that $y_a < 1 < y_b$, and $z_L(y) > 0$ for $y \in (y_a, y_b)$. Therefore, since at equilibrium both $z_K(y)$ and $z_L(y)$ must be positive, we shall only consider values of $y \in (Max\{y_a, y_c\}, y_b)$. Obviously, $H(y) = 0$ when $y = Max\{y_a, y_c\}$ or when $y = y_b$, and $H(y) > 0$ for $y \in (Max\{y_a, y_c\}, y_b)$. Further noting that $y_{*N} = 1 \in (Max\{y_a, y_c\}, y_b)$ and $\overline{H} = H(y_{*N} = 1)$, we see that, as $H(y)$ is continuous, the number of steady states must be even, unless $H'(y_{*N} = 1) = 0$. This situation is illustrated in Figure 3. Moreover, if the normalized steady state is a saddle, with $H'(y_{*N} = 1) < 0$ from Proposition 7 and satisfying conditions of Proposition 4, at least one other steady y_{*A}, with $y_{*A} < y_{*N} = 1$ and $H'(y_{*A}) > 0$, which is a source or a sink, must coexist with the saddle normalized steady state y_{*N}.

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